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Kindle Content Experience  
Amazon.com, Inc.

Data Mining – January 06, 2020

# Let's Get Greedy and Genetically Ensemble the Feature Space

This work was completed at the Dept. of Information Technology, NITK Surathkal, under the guidance of Dr. Nagamma Patil

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# Agenda

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Feature Space and  
Information

2

Research Gaps in  
Feature Selection?

3

Greedy Hybrid  
Ensemble with GA

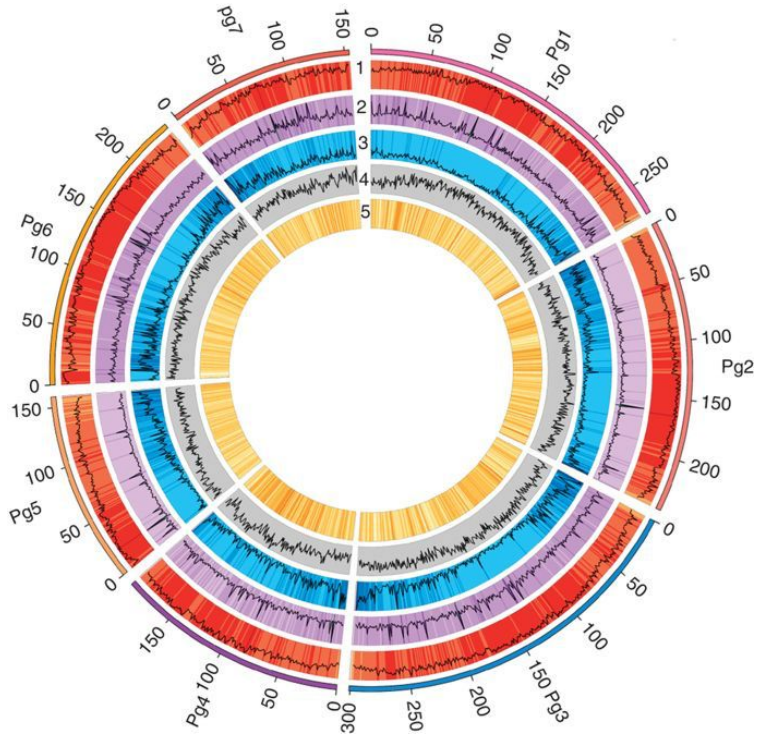
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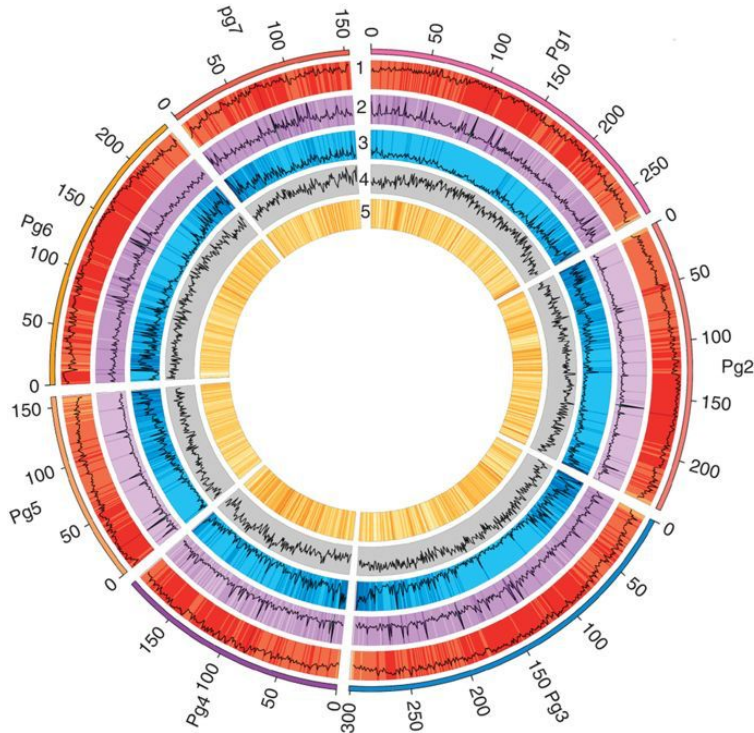
Conclusions and  
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# Feature Selection: Basic Idea



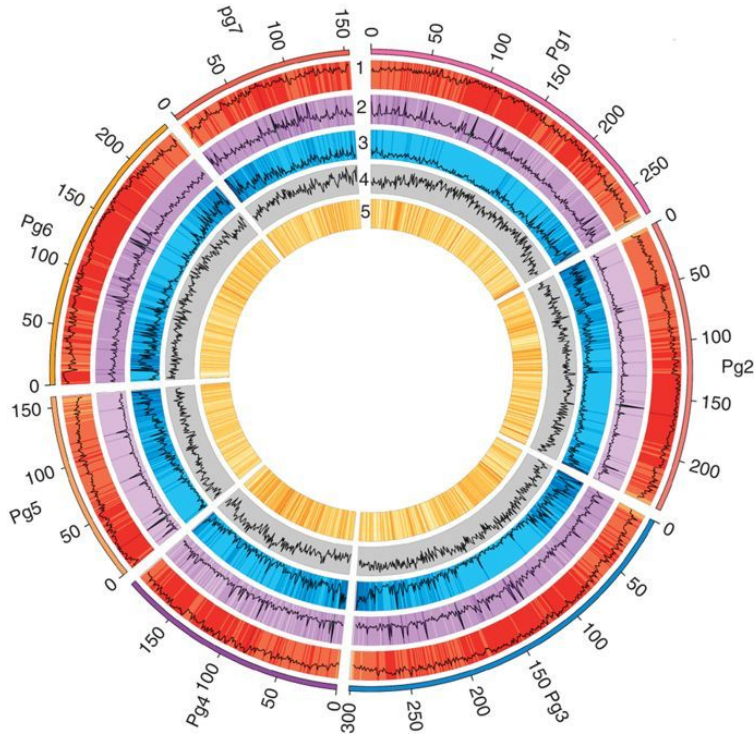
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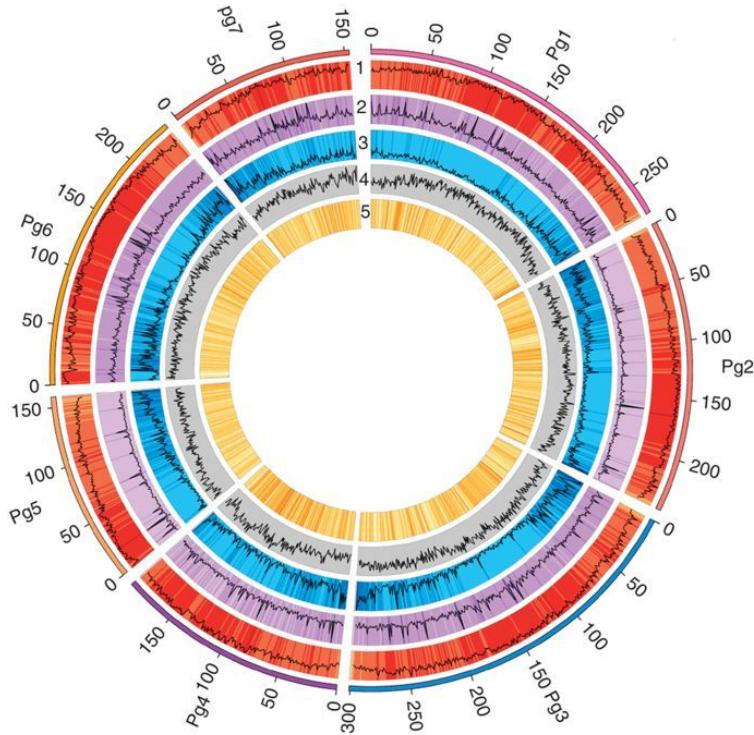
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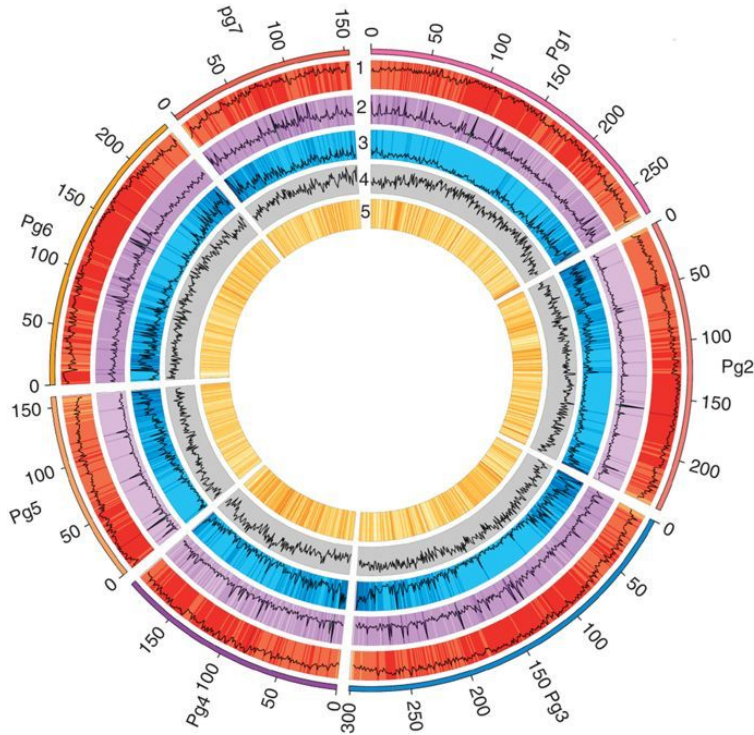
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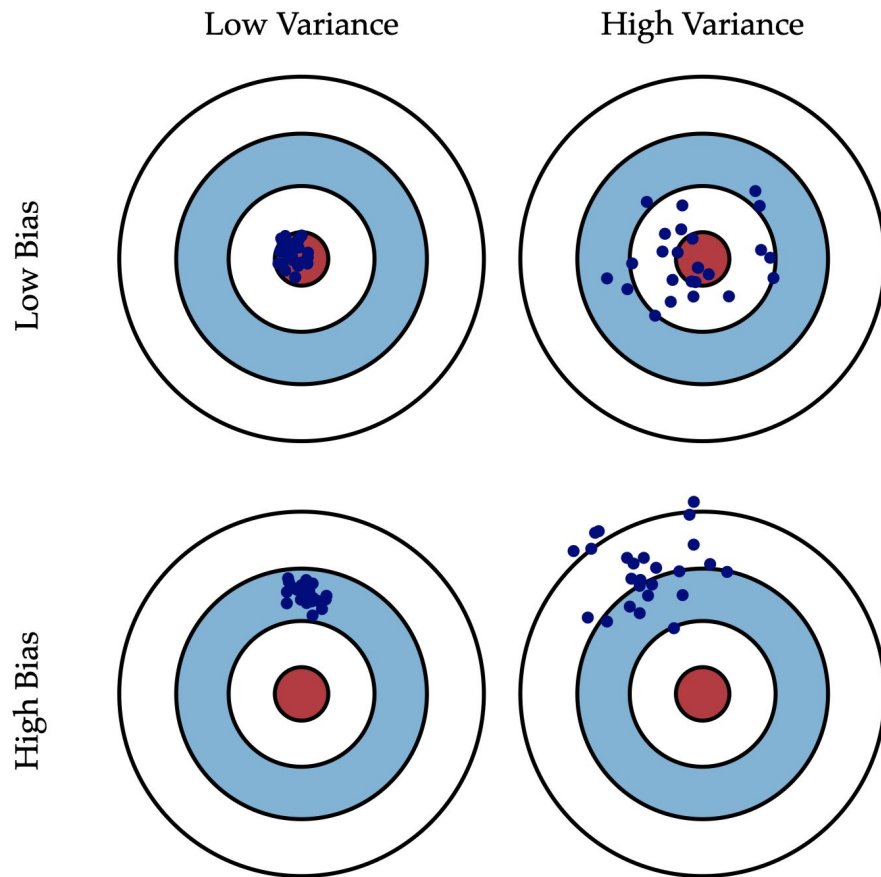
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- ❖ Bias-variance tradeoff – **bias = assumptions made by classifier; variance = training data variations**



# Bias-Variance Tradeoff

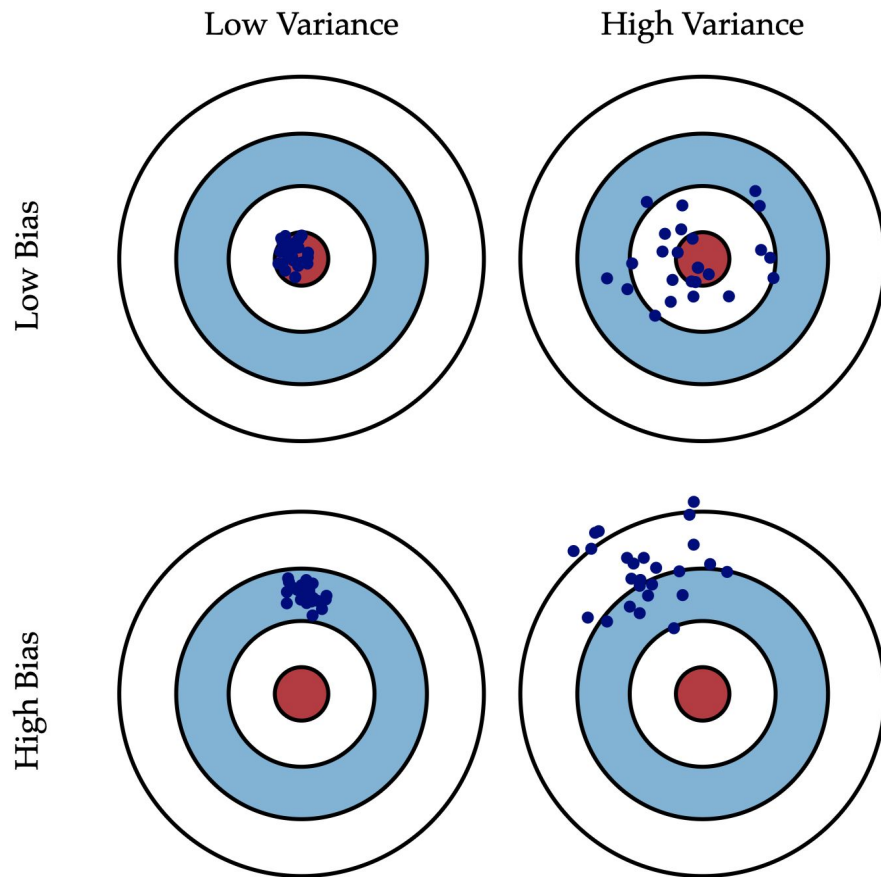
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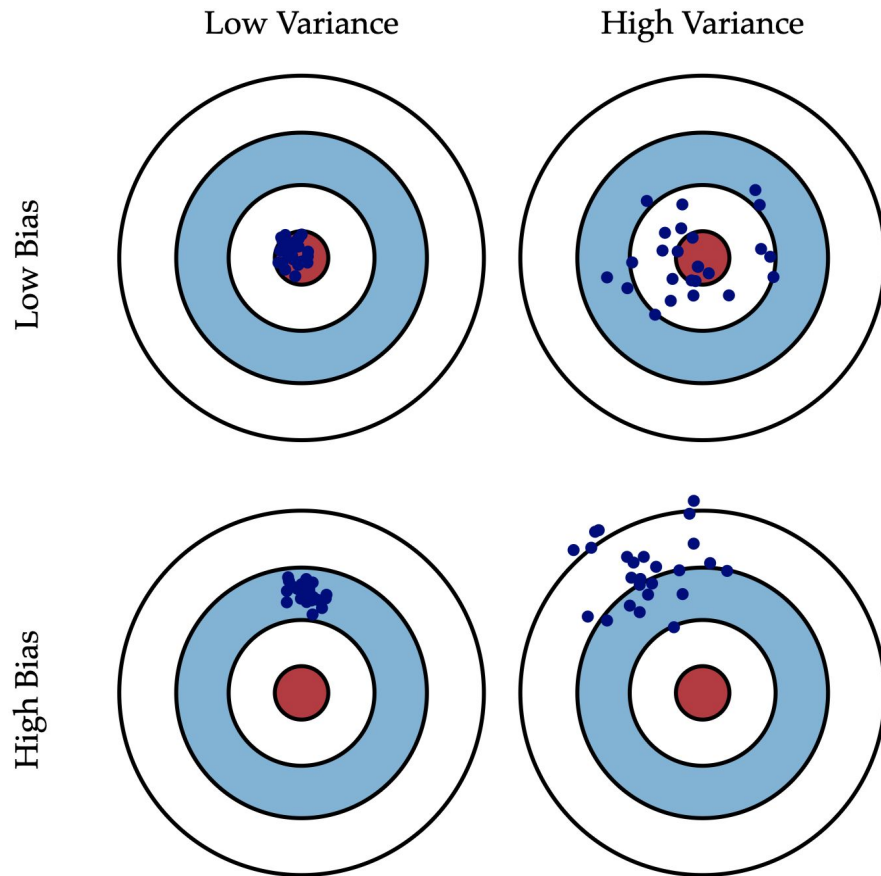
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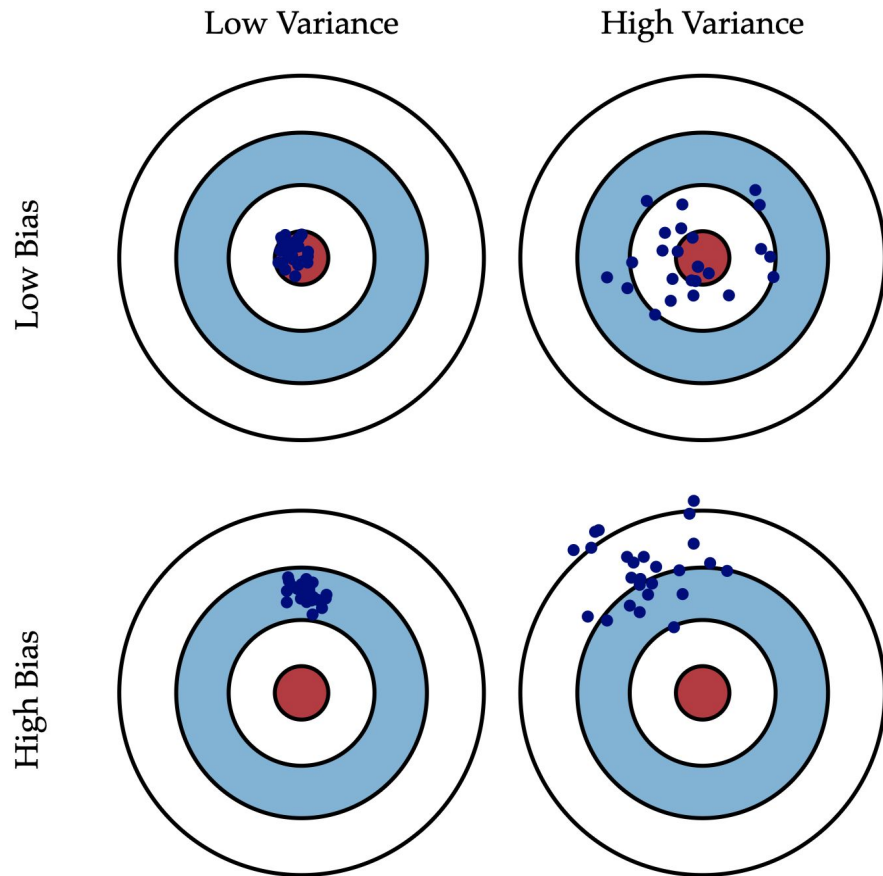
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- ❖ **Ideal:** features-to-samples ratio  $\ll 1$   
**Observed:** features-to-samples ratio  $\geq 1$



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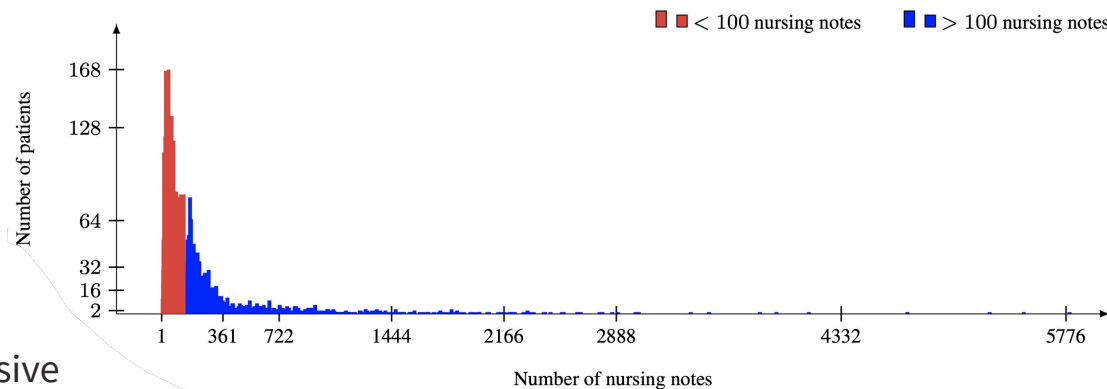
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# Feature Space and Information Extraction

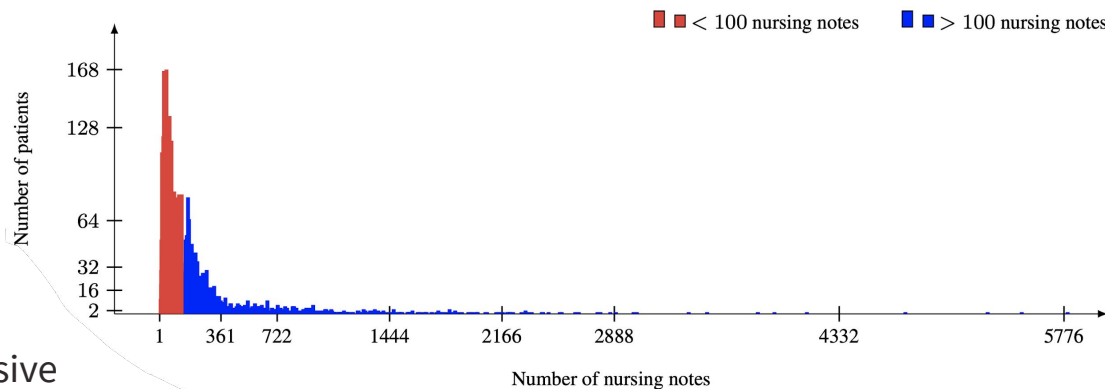
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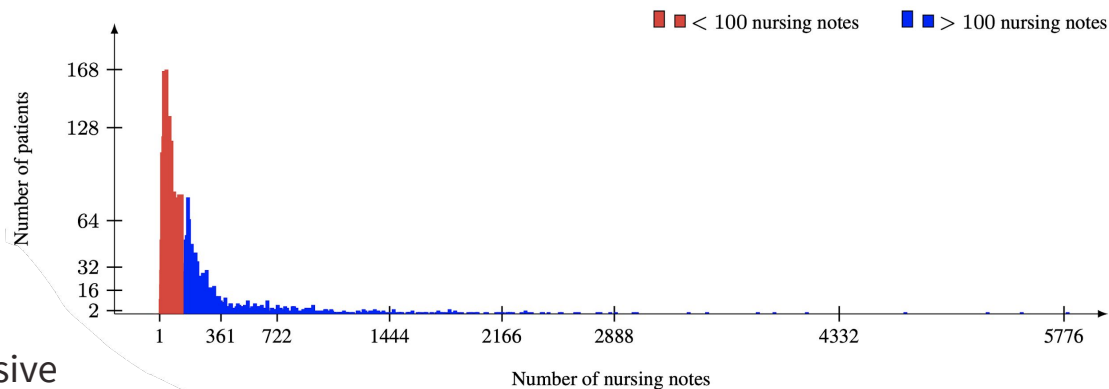
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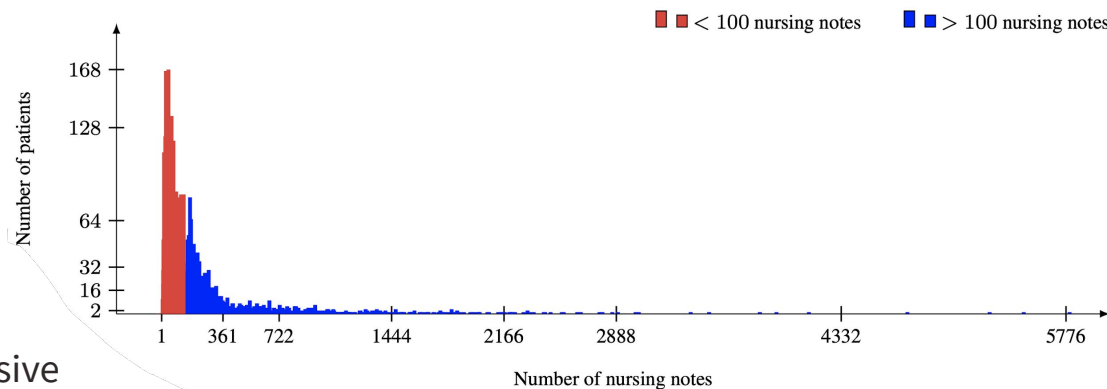
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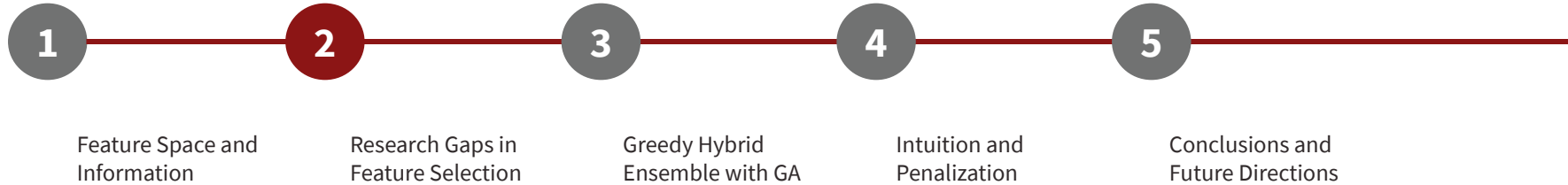
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- ❖ **Determine heuristically**: issue of convergence

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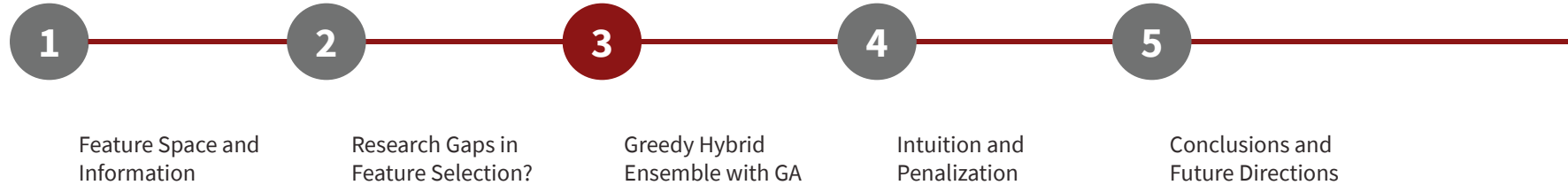
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- ❖ **Time and accuracy tradeoff:** use a hybrid of filter and wrapper approaches

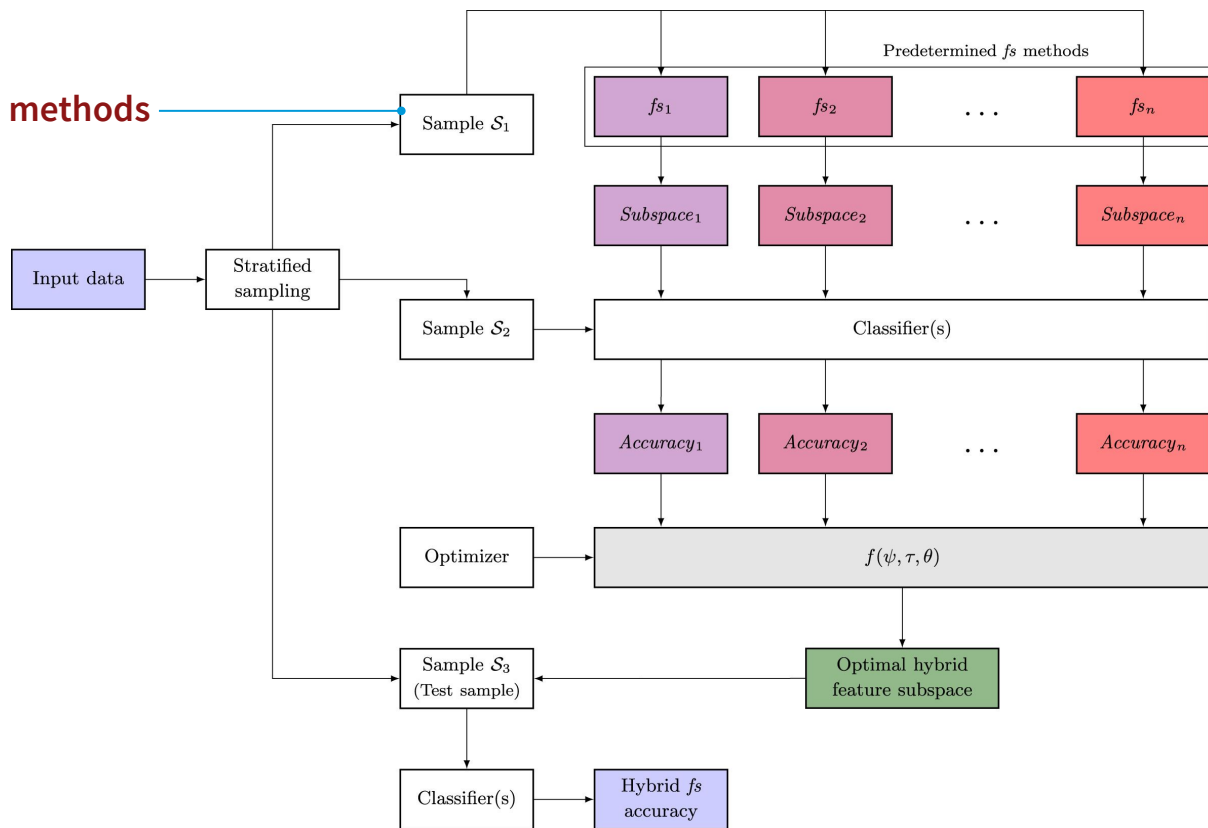
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# Greedy Filter–Wrapper Hybrid Ensemble

Feature selection using the chosen methods

Feature space: #features(dataset)



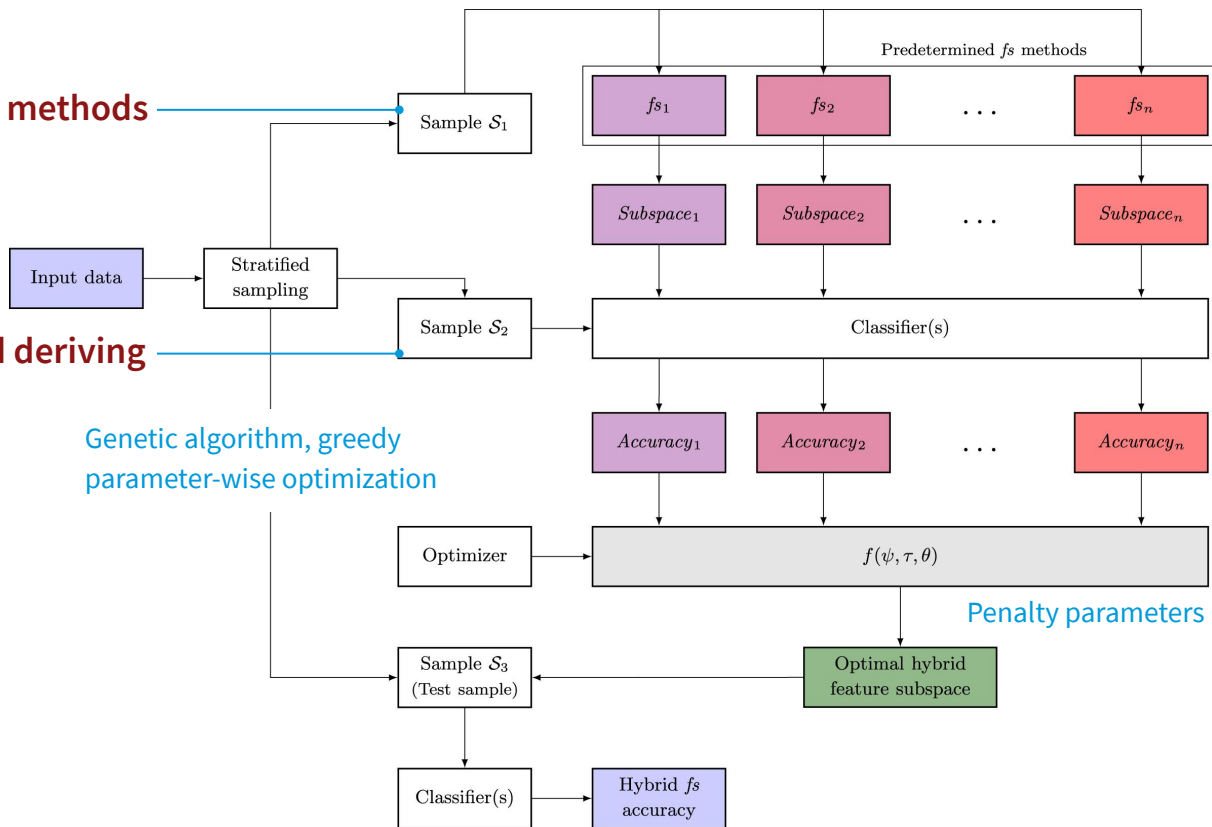
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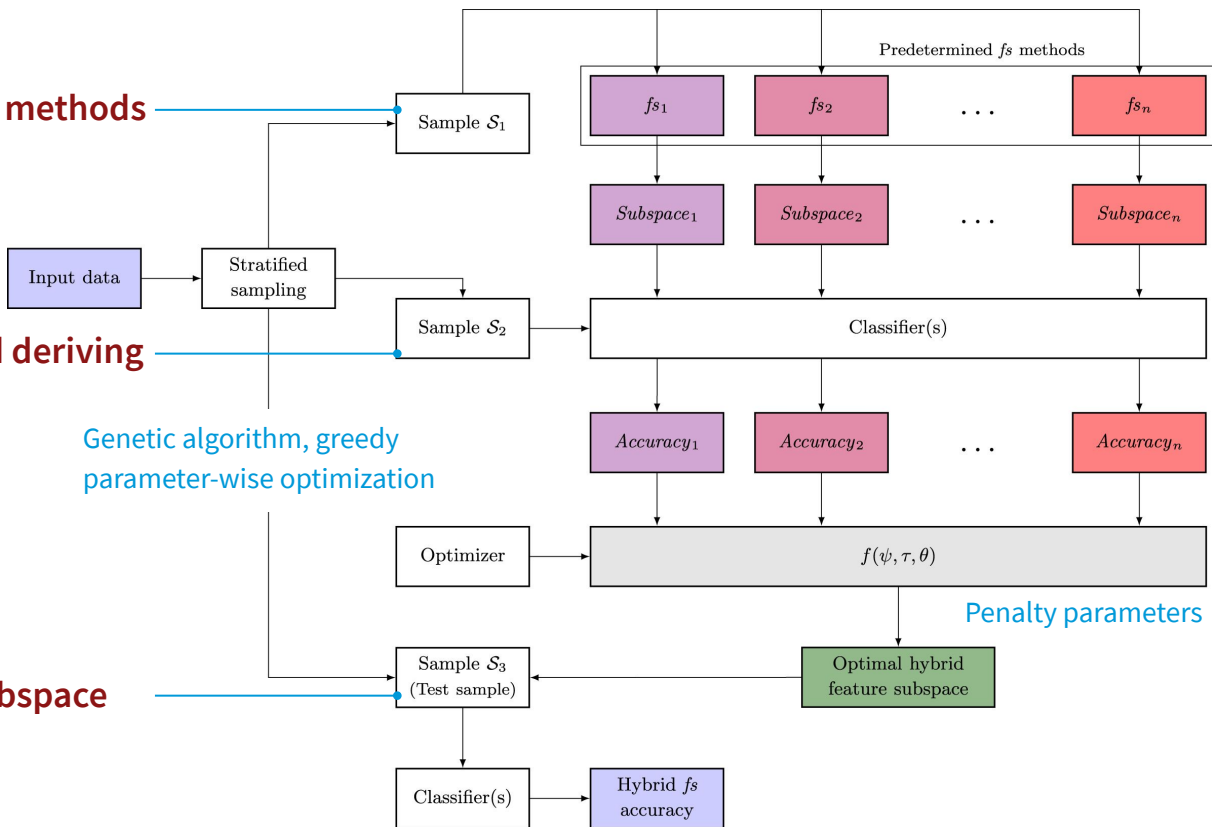
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**Evaluation of the hybrid feature subspace**

**Feature space:** hybrid



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# Greedy Hybrid Ensemble: Scoring Scheme

- ❖ Scoring of features (`featScore`) and selection techniques (`accScore`)


$$\text{featScore} = \begin{cases} \frac{|\text{FS}| - \rho_f + 1}{|\text{FS}|} & f \in \text{ranked FS} \\ 1/|\text{FS}| & f \in \text{unranked FS} \\ -1/|\text{FS}| & f \notin \text{FS} \end{cases}$$

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*(Note: In the original image,  $\rho_m$  is circled in red with an arrow pointing to the text "index(m) + 1".)*

- ❖ **Penalty parameters for greedy ensembling** of base feature subspaces

- ❖ **Accuracy penalty** ( $\psi$ ): reduces the impact of accuracy scores = `accScore`/ $\psi$
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$$\text{overallScore} = \sum_m^M \text{featScore}(f) \times \text{accScore}(m) \longrightarrow \text{Threshold-based feature selection}$$

General trends in parameter optimization: lower value of  $\psi$ , higher value of  $\tau$ , and fine tuning of  $\theta$

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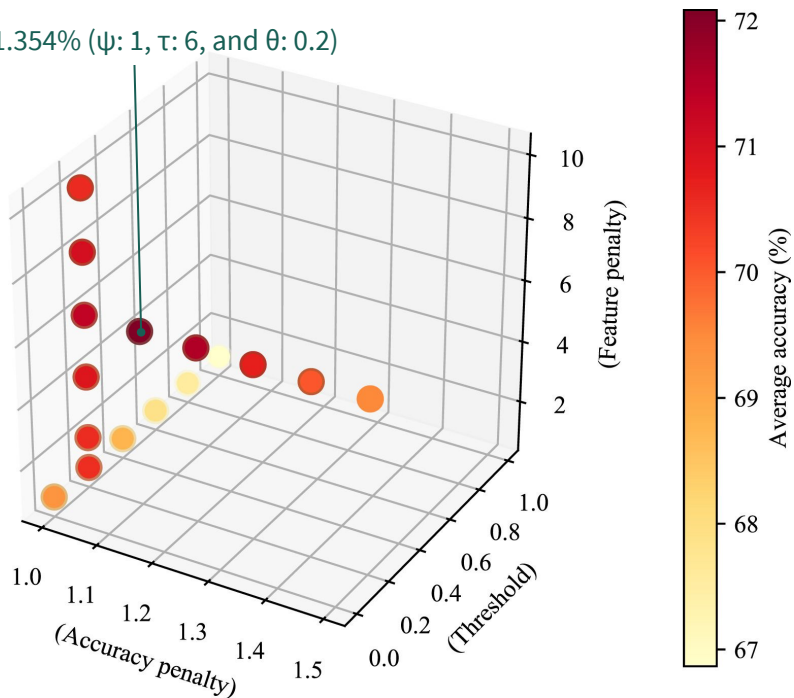
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- ❖ **Optimization of penalty parameters** ( $\psi$ ,  $\tau$ , and  $\theta$ ): genetic algorithm, greedy optimization, ...

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# Optimization of Penalty Parameters

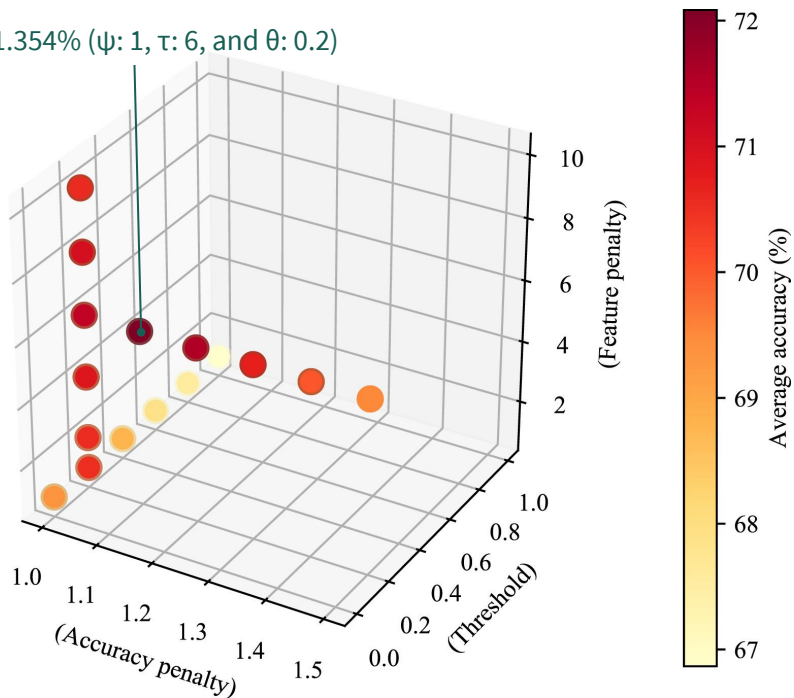
71.354% ( $\psi: 1, \tau: 6,$  and  $\theta: 0.2$ )



The effect of  $\psi, \tau,$  and  $\theta$  on Skin Cancer dataset  
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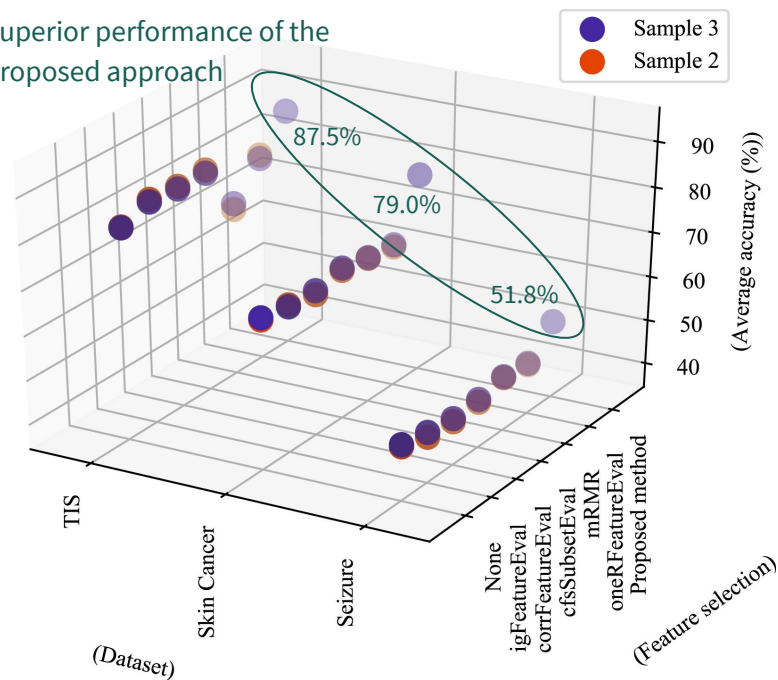
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Superior performance of the proposed approach



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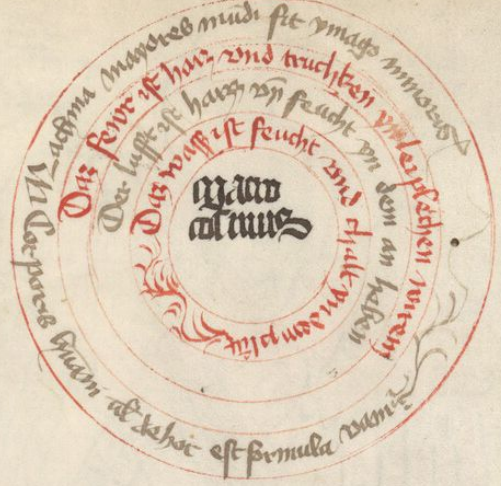
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- ❖ The proposed method introduces additional (penalty) parameters which **require prior training** to obtain the optimal setting in advance

# Further Reading

- [1] Gangavarapu, Tushaar, and Nagamma Patil. *A novel filter-wrapper hybrid greedy ensemble approach optimized using the genetic algorithm to reduce the dimensionality of high-dimensional biomedical datasets*. Applied Soft Computing (2019): 105538.
- [2] Tu, Qiang, Xuechen Chen, and Xingcheng Liu. *Multi-strategy ensemble grey wolf optimizer and its application to feature selection*. Applied Soft Computing 76 (2019): 16-30. Accessible: [sciencedirect/science/article/pii/S1568494618306793](https://doi.org/10.1016/j.asoc.2019.06.048).
- [3] Min, Fan, Qinghua Hu, and William Zhu. *Feature selection with test cost constraint*. International Journal of Approximate Reasoning 55.1 (2014): 167-179.
- [4] Dong, Hongbin, et al. *A novel hybrid genetic algorithm with granular information for feature selection and optimization*. Applied Soft Computing 65 (2018): 33-46. Accessible: [sciencedirect/science/article/pii/S1568494618300048](https://doi.org/10.1016/j.asoc.2018.06.048).
- [5] Masood, Mustafa K., Yeng Chai Soh, and Chaoyang Jiang. *Occupancy estimation from environmental parameters using wrapper and hybrid feature selection*. Applied Soft Computing 60 (2017): 482-494.
- [6] Chandrashekar et al. *A survey on feature selection methods*. Computers & Electrical Engineering 40.1 (2014).

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**G**regorius in dem gehente puche  
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wegerug anhangent dem oberste dñ-  
gen vñ auch ist er die helle vñ er  
mit seiner choug sich selb berubt  
mit den vñ dñsten vñ star muss Er  
ist auch der erweid das do mit guttñ  
werck mit guter hoffnung feucht  
vñ getet Er ist dar mer der do mett  
licher sacht p dñmet vñ das nett  
mit seiner vnstet ragende ist als au  
gustmñ spricht in dem xv puche  
von der stat plauus der ander vñ  
gelactist mensli lemen der mögñ  
der werlt lauff vñ lebes natur  
volgent ist vñ mer das fleische des  
menslich genaget ist vñ d' werlt //

Thank you ~